Find the limits

① $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - 1}{x}$

② $\lim\limits_{x \to 0} \dfrac{\sqrt{1+x} - 1 - \frac{x}{2}}{x^2}$

$$\lim_{x \to 0} \frac{\sqrt{1+x} - 1}{x}$$

$.00001$

$-.00001$

$$\frac{\sqrt{1+.00001} - 1}{.00001} = .4\bar{9}$$

$$\frac{\sqrt{1-.00001} - 1}{-.00001} = .5000$$

$$\lim_{x \to 0} f(x) = .5$$

$$\lim_{x\to 0} \frac{\sqrt{1+x}-1}{x} \qquad \frac{\sqrt{1+x}-1}{x}\cdot\frac{\sqrt{1+x}+1}{\sqrt{1+x}+1} = \frac{(1+x)-1}{x\sqrt{1+x}+x}$$

$$\frac{x}{x\sqrt{1+x}+x} = \frac{1}{\sqrt{1+x}+1} \qquad \lim_{x\to 0}\frac{1}{\sqrt{1+x}+1} = \frac{1}{\sqrt{1+0}+1} = \frac{1}{2}$$

$$\lim_{x\to a}\frac{f(x)}{g(x)} = \frac{f'(a)}{g'(a)}$$

L'Hopital's Rule

$$f(x) = \sqrt{1+x} - 1$$

$$g(x) = x$$

$$f'(x) = \frac{1}{2\sqrt{1+x}} \qquad \frac{1}{2\sqrt{1+0}} = \boxed{\frac{1}{2}}$$

$$g'(x) = 1$$

L'Hopital's Rule
stronger form

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)} = \frac{f''(a)}{g''(a)}$$

#2 on warm-up

$$f(x) = \sqrt{1+x} - 1 - \frac{x}{2}$$

$$g(x) = x^2$$

$$g'(x) = 2x$$

$$g''(x) = 2$$

$$f'(x) = \frac{1}{2\sqrt{1+x}} - \frac{1}{2}$$

$$f''(x) = -\frac{1}{4\sqrt{1+x}^3}$$

$$\Rightarrow \left. \frac{-\frac{1}{4\sqrt{1+x}^3}}{2} \right|_{x=0} = \frac{-\frac{1}{4}}{2} = -\frac{1}{8}$$

$$\lim_{x \to 0} \frac{x + \sin x}{x}$$

$f(x) = x + \sin x$ $\qquad$ $f'(x) = 1 + \cos x$ $\qquad \Rightarrow \dfrac{1 + \cos 0}{1} = 2$

$g(x) = x$ $\qquad$ $g'(x) = 1$

Also works for $\frac{\infty}{\infty}$

$$\lim_{x \to \frac{\pi}{2}} \frac{\sec x}{1 + \tan x}$$

$$\frac{d}{dx}\sec x = \sec x \tan x \qquad \frac{\cancel{\sec x}\tan x}{\sec^2 x}$$

$$\frac{d}{dx}\quad 1 + \tan x = \sec^2 x \qquad \frac{\tan x}{\sec x} = \cos x \tan x$$

$$\lim_{x \to \pi/2} \sin x = \boxed{1} \qquad \frac{\cancel{\cos x}}{1}\,\frac{\sin x}{\cancel{\cos x}} = \sin x$$

Sect. 8.2 # 1-8, 13-16

Dam Problem Due Friday (tomorrow)